Gravity-wave interferometers as quantum-gravity detectors
(title may be changed in the journal)

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A prediction of nearly all approaches to the unification of Quantum Mechanics and Gravity is that at very short distances the sharp classical concept of space-time should give way to a somewhat "fuzzy" (or "foamy") picture [1-4]. The properties of this fuzziness and the length scale characterizing its onset are potentially a means for determining which (if any) of the existing Quantum Gravity models is correct, but it has been generally believed [5] that the smallness of the quantum space-time effects would not allow to study them with presently available technologies. Here I observe that some proposals for the nature of this space-time fuzziness would affect the operation of gravity-wave interferometers by effectively introducing an additional source of noise that can be tightly constrained experimentally. In particular, I show that noise levels recently achieved rule out values of the length scale that characterizes one of the fuzziness proposals down to the Planck length ($L_p \sim 10^{-35}m$) and beyond, while for another proposal Planck-level sensitivity is within reach of gravity-wave interferometers that will start operating in the near future.

In a fuzzy space-time the operative definition of a distance D is affected by quantum fluctuations. These fluctuations are primarily characterized by their overall magnitude σ_D (the root-mean-square deviation of D). The simplest proposals are such that

$$\sigma_D \geq L_{min} , \tag{1}$$

where L_{min} is a Quantum-Gravity scale expected to be simply related to (usually identified with) the Planck length. Relations of the type (1) are motivated by certain analyses of gedanken experiments (see, *e.g.*, Ref. [6]) that combine some elements of Quantum Mechanics and some elements of Gravity. The quantum space-time fluctuations responsible for (1) are often visualized as involving geometry and topology fluctuations [1], virtual black holes [4], and other novel phenomena.

Other scenarios for space-time fuzziness arise when taking into account the quantum properties of devices, which were ignored in the original studies [6] that led to the proposal of Eq. (1). It is well understood (see, *e.g.* Refs. [7-12]) that the combination of the gravitational properties and the quantum properties of devices can have an important role in the analysis of the operative definition of gravitational observables.

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The implications can be far-reaching; in particular, in Ref. [10] it was observed that the masses of the probes used in measurements induce a change in the space-time metric and this is associated to the emergence of nonlocality. The nature of the gravitationally-induced nonlocality suggests [10] a modification of the fundamental commutators.

Here I shall be primarily concerned with the role that the quantum and the gravitational properties of devices have in the analysis of the measurability of distances, in the sense first illustrated in an influential study by Wigner [13]. Wigner derived a quantum limit on the measurability of the distance D separating two bodies by analysing a measurement procedure based on the exchange of a light signal between the bodies. Taking into account Heisenberg's position-momentum uncertainty relations also for the clock used in the measurement procedure Wigner obtained a lower bound on the quantum uncertainty in D:

$$\delta D \geq \sqrt{\frac{\hbar T_{obs}}{2 M_c}} \sim \sqrt{\frac{\hbar D}{c M_c}} \ , \tag{2}$$

where M_c is the mass of the clock, T_{obs} is the time required by the measurement procedure, and on the right-hand-side I used the fact that the Wigner measurement of a distance D requires a time $2D/c$.

The result (2) may at first appear somewhat puzzling, since ordinary Quantum Mechanics should not limit the measurability of any given observable. [It only limits the combined measurability of pairs of conjugate observables.] However, Quantum Mechanics is the theoretical framework for the description of the outcome of experiments performed by classical devices. In the limit in which the devices (*e.g.* Wigner's clock) behave "classically", which in particular requires the devices to be infinitely massive (so that $\delta x \, \delta v \sim \hbar/m \sim 0$), the right-hand side of equation (2) tends to zero. Therefore, as expected, there is no limitation on the measurability of the distance D in the appropriate infinite-mass "classical-device limit." This line of argument depends crucially on the fact that ordinary Quantum Mechanics does not involve Gravity.

Quite clearly the classical infinite-mass limit is not consistent with the nature of measurements involving gravitational effects. As the devices get more and more massive they increasingly disturb the gravitational/geometrical observables, and well before reaching the infinite-mass limit the procedures for the measurement of gravitational observables cannot be meaningfully performed [9, 11, 12]. A well-known example of this problem has been encountered in attempts (see, *e.g.*, Ref. [7]) to generalize to the study of the measurability of gravitational fields the famous Bohr-Rosenfeld analysis [14] of the measurability of the electromagnetic field. In order to achieve the accuracy allowed by the formalism of ordinary Quantum Mechanics, the Bohr-Rosenfeld measurement procedure resorts to ideal test particles of infinite mass, which would of course not be admissable probes in a gravitational context. Similarly, in Wigner's measurement procedure the limit $M_c \to \infty$ is not admissable when gravitational interactions are taken into account. At the very least the value of M_c is limited by the requirement that the clock should not turn into a black hole (which would not allow the required exchange of signals between the clock and the other devices). These observations, which render unavoidable the $\sqrt{T_{obs}}$-dependence of Eq. (2), provide motivation for the possibility [11, 12] that in Quantum Gravity any measurement that monitors a distance D for a time T_{obs} is affected by quantum fluctuations characterized by

$$\sigma_D \sim \sqrt{L_{QG} \, c \, T_{obs}} \ , \tag{3}$$

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where L_{QG} is a fundamental length scale which we can expect to be simply related to the Planck length. In particular, the Wigner measurement of a distance D, which requires a time $2D/c$, would be affected by fluctuations of magnitude $\sqrt{L_{QG}D}$. [Interestingly, the study reported in Ref. [8] analyzed the interplay of Quantum Mechanics and Gravity in defining a net of time-like geodesics and suggested that the maximum "tightness" achievable for the geodesics net is $\sqrt{L_{QG}D}$.]

A σ_D that increases with T_{obs} is not surprising for space-time fuzziness scenarios; in fact, the same phenomena that would lead to fuzziness are also expected to induce "information loss" [4] (the information stored in a quantum system degrades as T_{obs} increases). The argument based on the Wigner setup provides motivation to explore the specific form $\sigma_D \sim \sqrt{T_{obs}}$ of this T_{obs}-dependence.

From the type of T_{obs}-dependence of Eq. (3), and the stochastic properties of the processes [1-4] expected to characterize a fuzzy space-time, it follows that the quantum fluctuations responsible for (3) should have amplitude spectral density $S(f)$ with the f^{-1} dependence typical of "random walk noise" [15]:

$$S(f) = f^{-1}\sqrt{L_{QG}\, c}\; . \tag{4}$$

If indeed $L_{QG} \sim 10^{-35} m$, from (4) one obtains $S(f) \sim f^{-1} \cdot (5 \cdot 10^{-14} m\sqrt{Hz})$. [Of course, one expects that this formula for the Quantum-Gravity induced $S(f)$ could only apply to frequencies f significantly smaller than the Planck frequency c/L_p and significantly larger than the inverse of the time scale over which, even ignoring the gravitational field generated by the devices, the classical geometry of the space-time region where the experiment is performed manifests significant curvature effects.]

Before commenting on how the proposal (3)-(4) compares with data from modern gravity-wave interferometers, let me consider another space-time fuzziness scenario, which involves fluctuations of significantly different magnitude. This alternative scenario [9, 16] is essentially based on the observation that the uncertainty described by Wigner's Eq. (2) can be combined with a classical-Gravity estimate of the uncertainty in the measurement of the distance D that results from the distorsion of geometry associated to the gravitational field generated by the clock. While the uncertainty (2) dicreases with M_c, the uncertainty induced by Gravity increases with M_c, and combining the two uncertainties one finds a minimum total uncertainty of the type $\delta D \sim (\mathcal{L}_{QG}^2\, c\, T_{obs})^{1/3}$, where $\mathcal{L}_{QG}$ is a length scale analogous to L_{QG}. Just like in the other analyses of the dependence of the Wigner measurement on the time of observation, one is then led to consider a fuzzy space-time with corresponding T_{obs}-dependence:

$$\sigma_D \sim (\mathcal{L}_{QG}^2\, c\, T_{obs})^{1/3}\; . \tag{5}$$

The associated amplitude spectral density is

$$\mathcal{S}(f) = f^{-5/6}(\mathcal{L}_{QG}^2\, c)^{1/3}\; , \tag{6}$$

which for $\mathcal{L}_{QG} \sim 10^{-35} m$ gives $\mathcal{S}(f) = f^{-5/6} \cdot (3 \cdot 10^{-21} m\, Hz^{1/3})$.

Each of the proposals (1), (3), (5) was obtained within a corresponding scheme for the interplay between Quantum Mechanics and weak-field Gravity. This is an approach that has already proven successful in Quantum-Gravity research; in fact, the

phenomenon of gravitationally induced phases, which was also predicted from analyses of the interplay between Quantum Mechanics and weak-field Gravity, has already been confirmed experimentally [17]. By discovering experimentally which of the space-time fuzziness proposals is correct, we could also obtain additional insight in the weak-field limit of Quantum Gravity, and the requirement of consistency with the correct weak-field limit can represent a highly non-trivial constraint for the search of Quantum Gravity. In particular, the very popular Quantum-Gravity theories based on *Critical Strings* appear to require fuzziness of type (1), as seen in analyses of string collisions at Planckian energies [18], and a proposal for a quantum-group structure which might accommodate (1) has been discussed in Ref. [19]. The proposal (3) has been found to arise within the mathematical framework of dimensionfully deformed Poincaré symmetries[20, 21], which has been attracting much interest recently. Point-particle Quantum-Gravity theories based on these deformations would therefore require space-time fuzziness of type (3). Moreover, Eq. (3) has been shown to hold within Liouville (*non-critical*) String Theory [3, 22], another approach to Quantum Gravity which is attracting significant interest. The search of Quantum-Gravity theories whose weak-field limit is consistent with (5) has not yet been successful, but there appears to be no in principle obstruction and therefore one can expect progress in this direction to be forthcoming.

While conceptually the proposals (1), (3) and (5) represent drastic departures from conventional physics, phenomenologically they appear to encode only minute effects; for example, it has been observed that, assuming L_{min}, L_{QG} and $\mathcal{L}_{QG}$ are not much larger than the Planck length, all of these proposals encode submeter fluctuations on the size of the whole observable universe (about 10^{10} light years). However, the precision [23] of modern gravity-wave interferometers is such that they can provide significant information at least on the proposals (3) and (5). In fact, the operation of gravity-wave interferometers is based on the detection of minute changes in the positions of some test masses (relative to the position of a beam splitter). If these positions were affected by quantum fluctuations of the type discussed above the operation of gravity-wave interferometers would effectively involve an additional source of noise due to Quantum-Gravity. This observation allows to set interesting bounds already using existing noise-level data obtained at the *Caltech 40-meter interferometer*. This interferometer has achieved [24] displacement noise levels with amplitude spectral density lower than $10^{-18} m/\sqrt{Hz}$ for frequencies between 200 and 2000 Hz and this, as seen by straightforward comparison with Eq. (4), clearly rules out all values of L_{QG} down to the Planck length. Actually, even values of L_{QG} significantly lower than the Planck length are inconsistent with the data reported in Ref. [24]; in particular, by confronting Eq. (4) with the observed noise level of $3 \cdot 10^{-19} m/\sqrt{Hz}$ near 450 Hz, which is the best achieved at the *Caltech 40-meter interferometer*, one obtains the bound $L_{QG} \leq 10^{-40} m$. While at present we should allow for some relatively small factor to intervene in the relation between L_{QG} and L_p, having excluded all values of L_{QG} down to $10^{-40} m$ the status of the proposal (3) appears to be at best problematic. Of course, even more stringent bounds on L_{QG} are within reach of the next LIGO/VIRGO [25, 26] generation of gravity-wave interferometers.

The sensitivity achieved at the *Caltech 40-meter interferometer* also sets a bound on the proposal (5)-(6). By observing that Eq. (6) would imply Quantum-Gravity noise levels for gravity-wave interferometers of order $\mathcal{L}_{QG}^{2/3} \cdot (10 \, m^{1/3}/\sqrt{Hz})$ at frequencies of a few hundred Hz, one obtains from the data reported in Ref. [24] that $\mathcal{L}_{QG} \leq 10^{-29} m$.

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This bound is remarkably stringent in absolute terms, but is still well above the range of values of $\mathcal{L}_{QG}$ that is favoured by the intuition emerging from the various theoretical approaches to Quantum Gravity. A more significant bound on $\mathcal{L}_{QG}$ should be obtained by the LIGO/VIRGO generation of gravity-wave interferometers. For example, it is plausible [25] that the "advanced phase" of LIGO achieve a displacement noise spectrum of less than $10^{-20}m/\sqrt{Hz}$ near 100 Hz and this would probe values of $\mathcal{L}_{QG}$ as small as $10^{-34}m$.

Looking beyond the LIGO/VIRGO generation of gravity-wave interferometers, one can envisage still quite sizeable margins for improvement by optimizing the performance of the interferometers at low frequencies, where both (4) and (6) become more significant. It appears natural to perform such studies in the quiet environment of space, perhaps through future refinements of LISA-type setups [27].

The example of gravity-wave interferometers here emphasized shows that the smallness of the Planck length does not preclude the possibility of direct investigations of space-time fuzziness. This complements the results of the studies [28, 29] which had shown that indirect evidence of quantum space-time fluctuations could be obtained by testing the predictions of theories consistent with a given picture of these fluctuations. Additional encouragement for the outlook of experimentally-driven progress in the understanding of the interplay between Gravity and Quantum Mechanics comes from recent studies [30, 31] in the area of gravitationally induced phases, whose significance has been emphasized in Refs. [32, 33].

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